Imperial College Business School

Imperial means Intelligent Business

Instrumental Variables

Beating endogeneity…with music?

by Ralf Martin (r.martin@imperial.ac.uk)

Fixing endogeneity

Endogeneity Problem

Instrumental Variable idea:

$$Y = \beta X + \epsilon \quad \text{but} \quad E\{\epsilon | X\} \neq 0$$

$$X = X(A, B, Z)$$

- Many different factors are driving X
- Suppose there is at least one factor that is independent from ϵ:

$$E\{\epsilon | Z\} = 0$$

- We can then potentially use this variable to identify an unbiased estimate of β

2 Stage Least Squares Estimator (2 SLS)

Setup: $Y = \beta_0 + \beta X + \epsilon$

Instrument: Z

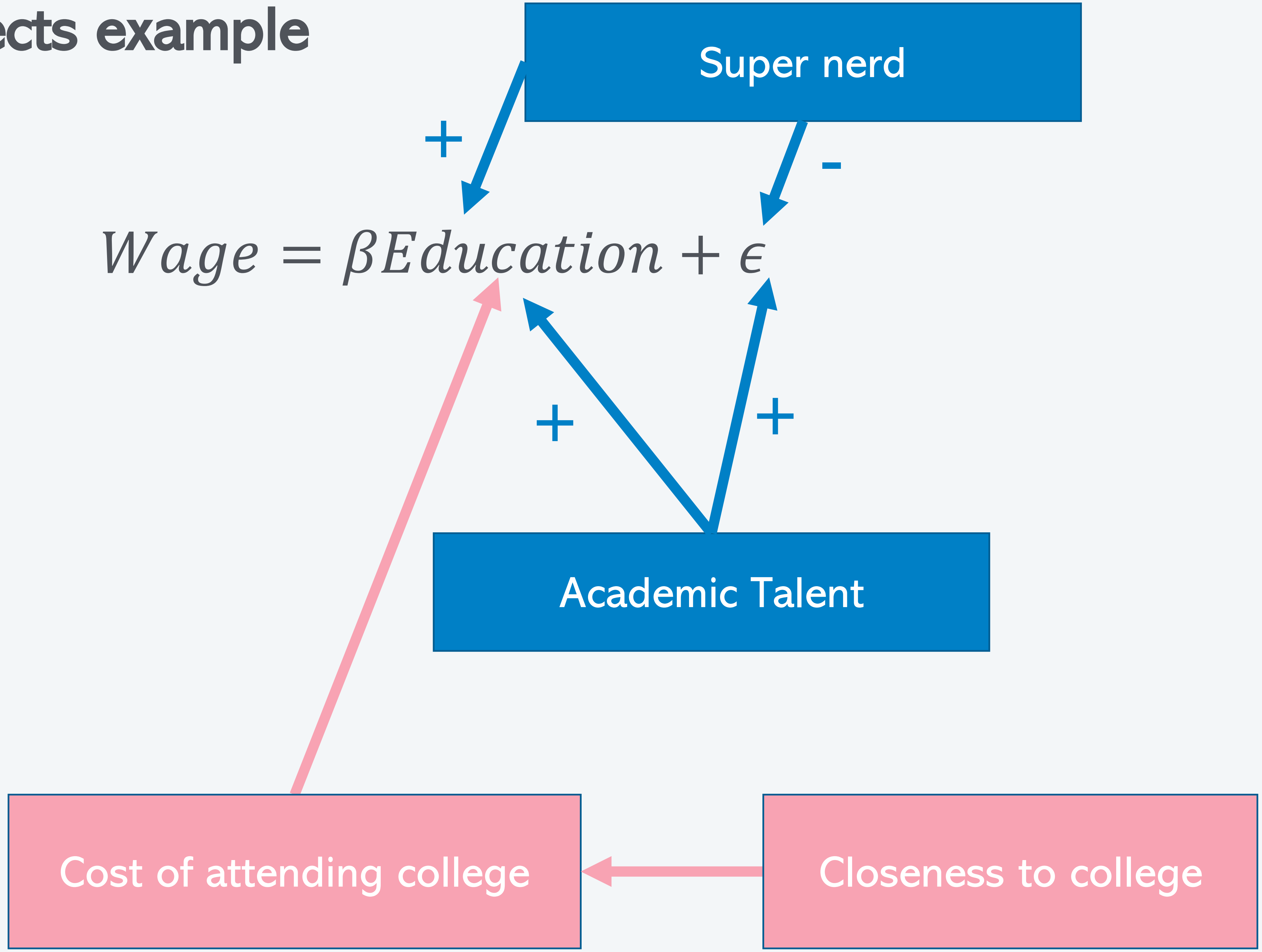
Stage 1: Regress X on Z; i.e. $X = \pi_0 + \pi Z + \eta$

Predict X: $\boldsymbol{\widehat{X} = \widehat{\pi}_0 + \widehat{\pi} Z}$

Stage 2: Regress Y on $\hat{X}$; i.e. $\mathrm{Y} = \beta_0 + \beta \hat{X} + \epsilon$

Unlike X, $\hat{X}$ is independent of ϵ because it is only driven by Z
Hence Stage 2 provides consistent (although not unbiased) estimate of β.

Schooling effects example



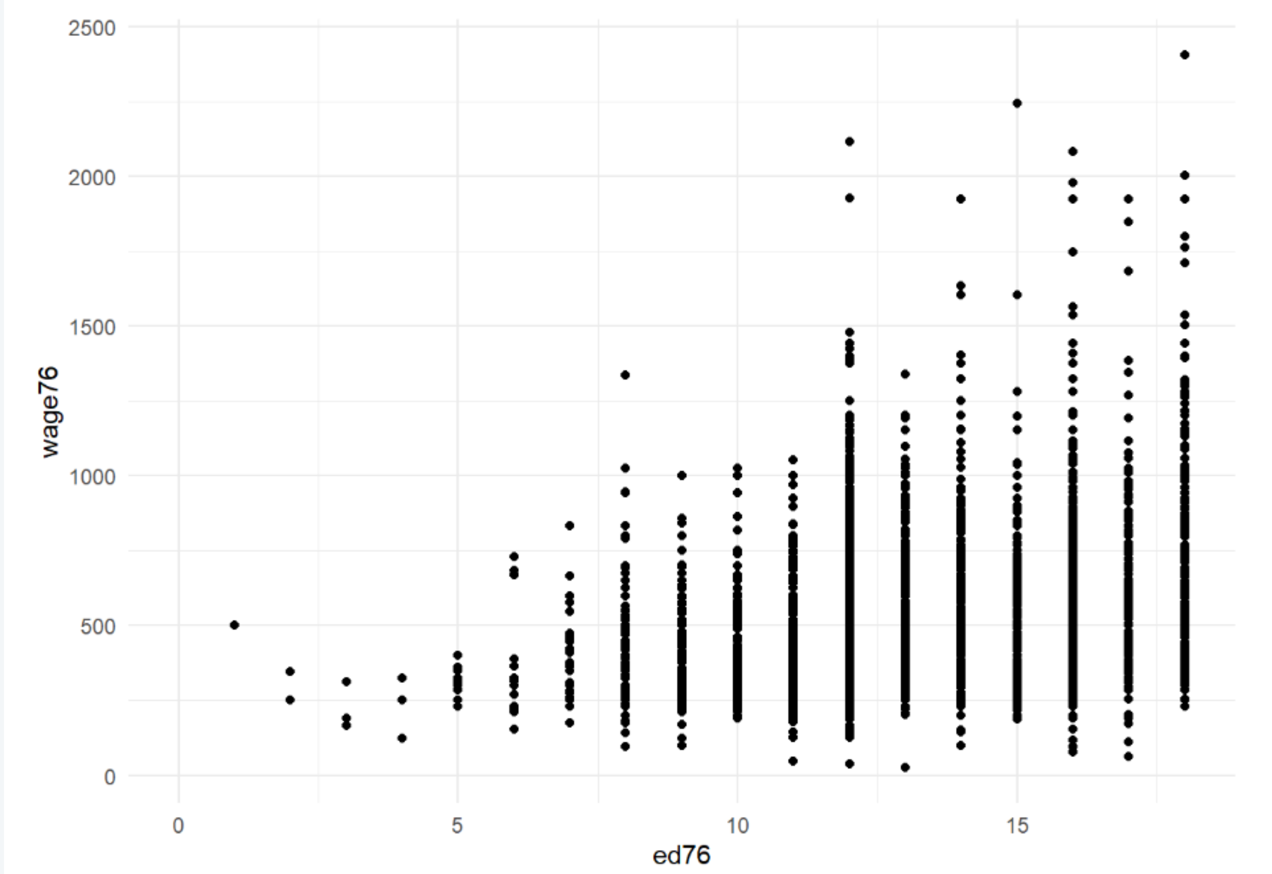
See also paper by Card 1993

Schooling example

```r
library(AER)
library(dplyr)
df=read.csv("https://www.dropbox.com/s/diecbkq03gfid0p/card1993.csv?dl=
1")
#from https://davidcard.berkeley.edu/data_sets.html

lm(wage76 ~ ed76, data = df) %>% summary()
```

```
##
## Call:
## lm(formula = wage76 ~ ed76, data = df)
##
## Residuals:
##     Min      1Q Median      3Q     Max
## -623.6 -173.7  -33.3  128.3 1687.9
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  183.934     23.160   7.942 2.78e-15 ***
## ed76          29.566      1.712  17.274  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '*      01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 251.7 on 3015 deg            dom
##   (596 observations deleted due to missingness)
## Multiple R-squared:  0.09006,    Adjuste
## F-statistic: 298.4 on 1 and 3015 DF,  p-
```



For every additional year of education $30 more

Schooling example: The distance instrument

```
first=lm(ed76~nearc4a, data = df)
first %>% summary()
```

```
##
## Call:
## lm(formula = ed76 ~ nearc4a, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.6575  -1.6575  -0.6575   2.3425   5.1935
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.80654    0.06343 201.911   <2e-16 ***
## nearc4a      0.85094    0.09041   9.412   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.717 on 3611 degrees of freedom
## Multiple R-squared:  0.02394,    Adjusted R-squared:  0.02367
## F-statistic: 88.58 on 1 and 3611 DF,  p-value: < 2.2e-16
```

- If you grow up near a public 4 year college you have nearly an additional year of education on average
- Note that this is super significant (including high F>10)

Implementing IV

New regression command to implement 2SLS [part of library(AER)]

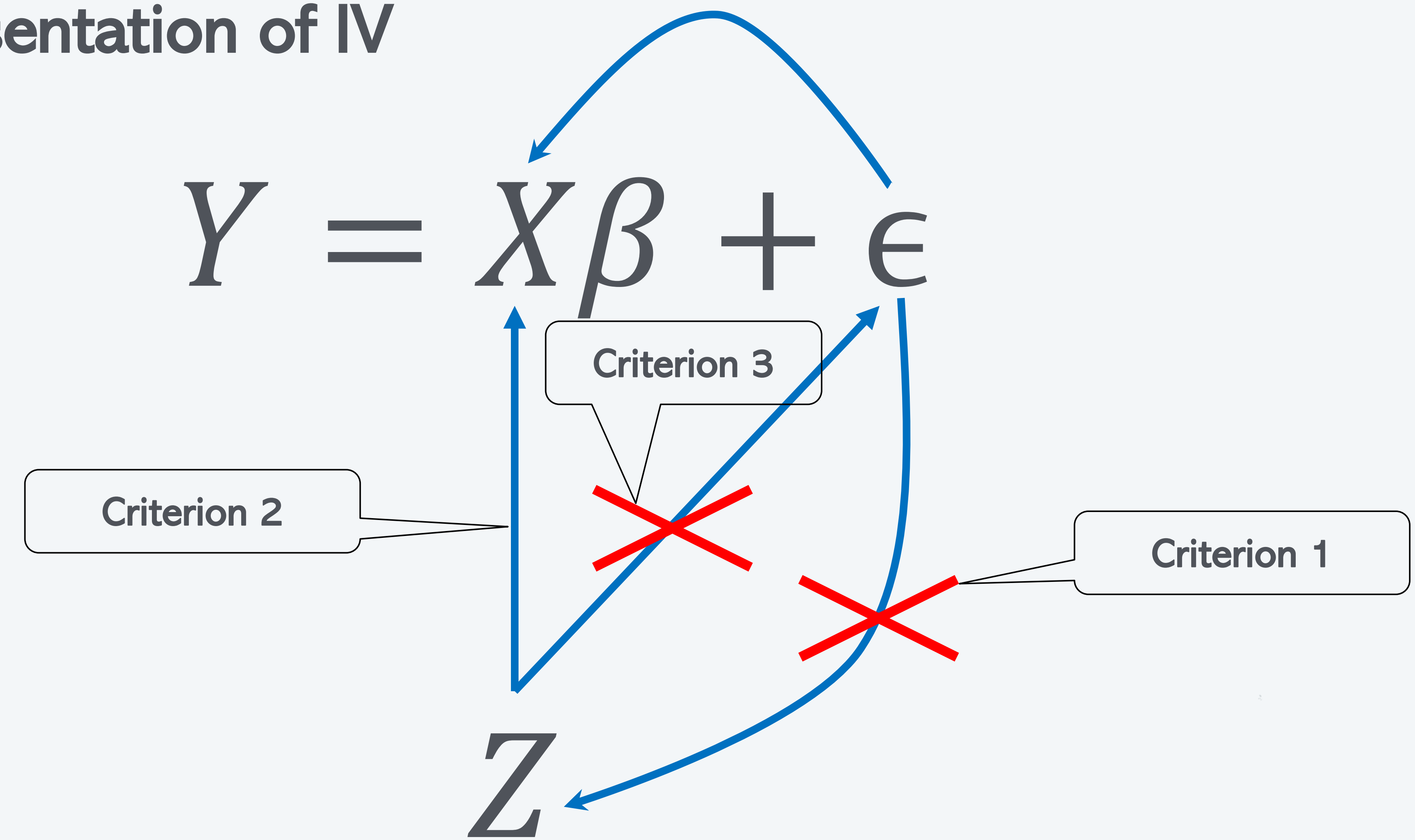
```
iv=ivreg(wage76 ~ ed76  | nearc4a, data=df)
iv %>% summary()
```

```
##
## Call:
## ivreg(formula = wage76 ~ ed76 | nearc4a, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -941.253 -211.663   -6.304  204.517 1683.696
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -943.77     198.71  -4.749 2.13e-06 ***
## ed76          114.59      14.97   7.652 2.64e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 339.4 on 3015 degrees of freedom
## Multiple R-Squared: -0.6547, Adjusted R-squared: -0.6552
## Wald test: 58.56 on 1 and 3015 DF,  p-value: 2.639e-14
```

Seems the effect of education is stronger than thought: an extra $114 for every year

Could mean
(a) In OLS case we had downward bias
(b) Could be indicative of problem with IV

A graphical representation of IV



Must be argued

Can be checked

3 Criteria for instrumental variables

1. Must be independent of shocks
2. Must be driver of variable of interest
3. Must not affect outcome variable other than trough variable of interest

Must be argued

The College distance instrument

$$Wage = EDU\beta + \epsilon$$

Criterion 3

Criterion 2

Criterion 1

Near College

Places with opportunities (e.g. cities)

Violation of criterion 3

Conditional IV

Conditioning/Control Variables e.g. controls for nice places

$$Wage = \beta EDU + \beta_2 X + \eta$$

Criterion 2

Criterion 3

Criterion 1

Near College

Places with opportunities (e.g. cities)

Conditional IV in action

```
iv=ivreg(wage76 ~ ed76+factor(region) +nearc4b+nearc2 | nearc4a+nearc4b
+nearc2+factor(region)
         , data=df )
iv %>% summary()
```

```
##
## Call:
## ivreg(formula = wage76 ~ ed76 + factor(region) + nearc
##     nearc4a + nearc4b + nearc2 + factor(region), data = d
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -853.100 -198.498   -6.864  187.265 1630.953
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       -729.039    291.987  -2.497  0.01258 *
## ed76                98.811     20.916   4.724 2.42e-06 ***
## factor(region)2  -126.976     40.111  -3.166  0.00156 **
## factor(region)3   -36.559     34.319  -1.065  0.28683
## factor(region)4   -21.313     42.946  -0.496  0.61974
## factor(region)5   -14.037     38.400  -0.366  0.71474
## factor(region)6   -62.001     30.296  -2.047  0.04079 *
## factor(region)7    17.031     24.201   0.704  0.48166
## factor(region)8   -11.016     23.908  -0.461  0.64499
## factor(region)9   -19.767     35.121  -0.563  0.57360
## nearc4b            -8.513     15.013  -0.567  0.57075
## nearc2             27.357     12.357   2.214  0.02691 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 311.4 on 3005 degrees of freedom
## Multiple R-Squared: -0.3884, Adjusted R-squared: -0.3935
## Wald test: 17.66 on 11 and 3005 DF,  p-value: < 2.2e-16
```

Conditioning/Control Variables e.g. controls for nice places

- Education coefficient remains high and significant

Checking first stage in conditional IV case

```
first=lm(ed76~nearc4a+factor(region) +nearc4b+nearc2, data = df)
first %>% summary()
```

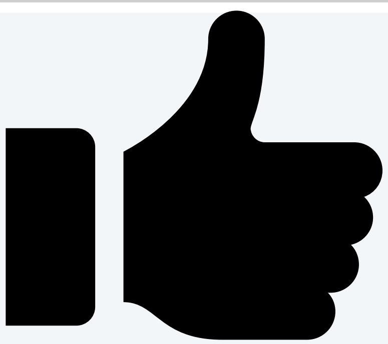
```
##
## Call:
## lm(formula = ed76 ~ nearc4a + factor(region) + nearc4b + nearc2,
##     data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.3438  -1.6767  -0.2662   2.0324   5.9761
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)     13.41923    0.18020  74.469  < 2e-16 ***
## nearc4a          0.66714    0.10785   6.186 6.88e-10 ***
## factor(region)2  0.25742    0.29739   0.866   0.3868
## factor(region)3 -0.88954    0.20580  -4.322 1.58e-05 ***
## factor(region)4 -1.39531    0.21202  -6.581 5.35e-11 ***
## factor(region)5 -1.31319    0.18230  -7.204 7.11e-13 ***
## factor(region)6 -0.17377    0.22849  -0.761   0.4470
## factor(region)7 -0.15300    0.18226  -0.839   0.4013
## factor(region)8 -0.15192    0.18456  -0.823   0.4105
## factor(region)9 -0.54352    0.25847  -2.103   0.0355 *
## nearc4b          0.25475    0.13183   1.932   0.0534 .
## nearc2           0.03420    0.09664   0.354   0.7234
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.662 on 3601 degrees of freedom
## Multiple R-squared:  0.06556,    Adjusted R-squared:  0.0627
## F-statistic: 22.97 on 11 and 3601 DF,  p-value: < 2.2e-16
```

IV still significant

- However we need a super strong IV
- Rule of thumb: F-stat of H0: Z=0 >10

```
linearHypothesis(first,c("nearc4a=0"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## nearc4a = 0
##
## Model 1: restricted model
## Model 2: ed76 ~ nearc4a + factor(region) + nearc4b + nearc2
##
##   Res.Df   RSS Df Sum of Sq      F    Pr(>F)
## 1   3602 25791
## 2   3601 25520  1    271.15 38.261 6.882e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Reduced form: regressing outcome on instrument

```
reduced=lm(wage76~nearc4a+factor(region)+nearc4b+nearc2, data = df)
reduced %>% summary()
```

```
##
## Call:
## lm(formula = wage76 ~ nearc4a + factor(region) + nearc4b + nearc2,
##     data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -631.80 -164.44  -37.61  120.43 1735.20
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      600.144     18.934  31.697  < 2e-16 ***
## nearc4a           65.301     11.192   5.835 5.97e-09 ***
## factor(region)2  -79.575     31.983  -2.488  0.01290 *
## factor(region)3 -122.686     21.488  -5.709 1.24e-08 ***
## factor(region)4 -152.045     22.014  -6.907 6.03e-12 ***
## factor(region)5 -142.303     19.136  -7.436 1.34e-13 ***
## factor(region)6  -72.354     24.341  -2.973  0.00298 **
## factor(region)7   -1.002     19.112  -0.052  0.95819
## factor(region)8  -25.757     19.240  -1.339  0.18075
## factor(region)9  -75.947     26.576  -2.858  0.00430 **
## nearc4b           18.110     13.632   1.329  0.18409
## nearc2            29.110      9.963   2.922  0.00351 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 252.2 on 3005 degrees of freedom
##   (596 observations deleted due to missingness)
## Multiple R-squared:  0.08978,    Adjusted R-squared:  0.08644
## F-statistic: 26.94 on 11 and 3005 DF,  p-value: < 2.2e-16
```

If reduced form is not significant IV won't be either

IV health warning

Don’t mess with weak instruments

- Estimator is consistent if the probability limit is equal to true parameter.
- Note 2SLS IV is consistent but not unbiasd

- **Properties of IV with a poor instrumental variable**
  - <u>IV may be much more inconsistent than OLS</u> if the instrumental variable is not completely exogenous and only weakly related to explanatory var x

$$\hat{\beta}_{OLS} \approx \beta + Corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

$$\hat{\beta}_{IV} \approx \beta + \frac{Corr(\epsilon, Z)}{\boxed{Corr(X, Z)}}\frac{\sigma_\epsilon}{\sigma_X}$$

There is no problem if the instrumental variable is really exogenous. If not, the bias will be the larger the weaker the correlation with x.

If $Corr(X, Z)$ is small = weak instrument IV could be more biased than OLS even if $Corr(\epsilon, Z) < Corr(\epsilon, X)$

That’s why we want the first stage F-statistic to be large!

Extra Slides

More instrumental variable examples

Additional controls many IV's example: Family size

$$Y = \beta FamSize + \epsilon$$

Are large families good or bad?

Outcome	Regression (1)
Highest Grade Completed	-0.145 (0.005)
Years of Schooling ≥ 12	-0.029 (0.001)
Some College (age ≥ 24)	-0.023 (0.001)
College Graduate (age ≥ 24)	-0.015 (0.001)

What do you think?

- Many potential confounders. Family size is by and large a choice
- Poorer and less educated parents tend to have more kids
- The same factors could drive outcomes

Family size and children outcomes

(Study by Angrist, Lavy & Schlosser)

However, there is randomness in family size as well

Two factors that are out of control of controlling parents:

- Occurrence of twins
- Sex of baby

Ok those might meet criteria 1 from earlier but how about criteria 2?

- Twins: families might only have planned for 2 kids, but when they had twins they un-intentionally had 3
- Many families have preference for a sex mix (a boy and a girl)
- Hence, if they have two kids of the same sex they are more likely to carry on having more kids

Second stage

		2SLS Estimates		
Outcome	Regression Estimates (1)	Twins Instruments (2)	Same-sex Instruments (3)	Twins & Samesex (4)
Years of Schooling	-0.145	0.174	0.318	0.237
	(0.005)	(0.166)	(0.210)	(0.128)
High School Graduate	-0.029	0.030	0.001	0.017
	(0.001)	(0.028)	(0.033)	(0.021)
Some College (for age ≥ 24)	-0.023	0.017	0.078	0.048
	(0.001)	(0.052)	(0.054)	(0.037)
College graduate (for age ≥ 24)	-0.015	-0.021	0.125	0.052
	(0.001)	(0.045)	(0.053)	(0.032)

Notes: This table reports OLS and 2SLS estimates of the effect of family size on schooling. OLS estimates appear in Column (1). Columns (2), (3) and (4) show 2SLS estimates constructed using the instruments indicated in column headings. Standard errors are reported in parentheses.

2SLS with multiple instruments & multiple endogenous explanatory variables

Setup: $Y = \beta_0 + \beta_1 X_2 + \beta_2 X_2 + \beta_c X_c + \epsilon$

Instruments: Z_1, Z_2,….

At least as many Z as endogenous X

Stage 1: Regress all X on all Z (and X_c); e.g.

$$X_1 = \pi_{01} + \pi_{11} Z_1 + \pi_{21} Z_2 + \pi_{c1} X_C + \eta_1$$
$$X_2 = \pi_{02} + \pi_{12} Z_1 + \pi_{22} Z_2 + \pi_{c2} X_C + \eta_2$$

Predict all endogenous X: $\boldsymbol{\hat{X}_1, \hat{X}_2}$

Stage 2: Regress Y on predicted endogenous X; i.e.

$$Y = \beta_0 + \beta_1 \hat{X}_2 + \beta_2 \hat{X}_2 + \beta_c \hat{X}_c + \epsilon$$

Another example: Subsidies and Unemployment

Some Causal Effects of an Industrial Policy

Chiara Criscuolo
Ralf Martin
Henry G. Overman
John Van Reenen

AMERICAN ECONOMIC REVIEW
VOL. 109, NO. 1, JANUARY 2019
(pp. 48-85)

Download Full Text PDF (Complimentary)

$$\Delta \ln Unemp = \beta \Delta NGE + \epsilon$$

Change in (log) unemployment between 2002 and 1997

Change in support rate

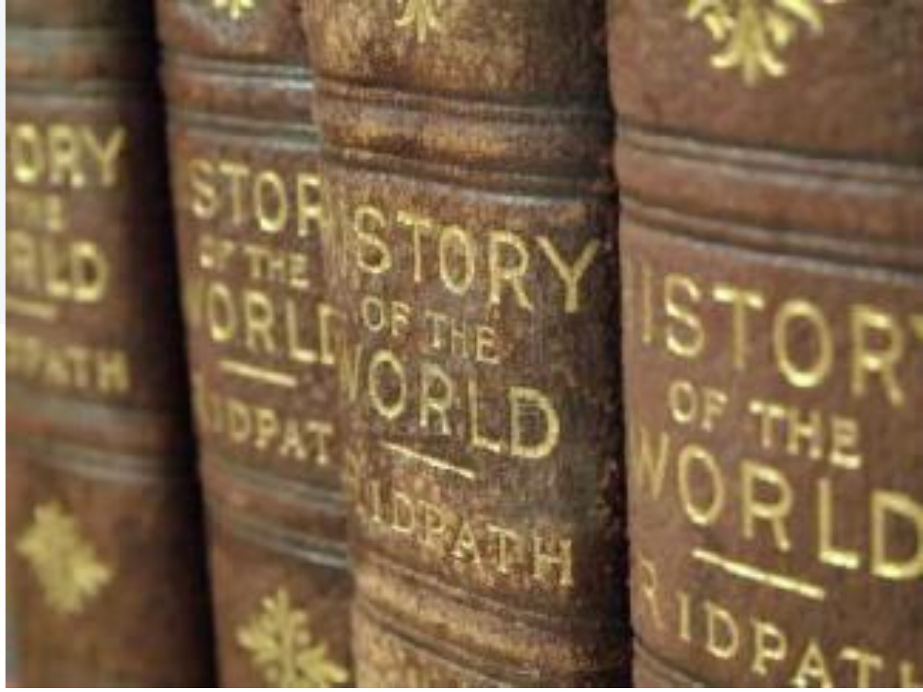
The 10 worst places to live in England revealed - and the results may surprise you!

23 July 2019, 13:04 | Updated: 23 July 2019, 15:49

The worst towns in the UK have been revealed in this 2019 survey. *Picture: Getty*

Instrument construction

Know your history



How much support an area gets is based on how

a) Various indicators of past deprivation (GDP, unemployment, etc.)
b) Weightings for different indicators

In reality not linear but let's use as approximation

$$NGE_t = f\left(D_t, W_{r(t)}\right) \approx \sum_k D_{kt} W_{kr(t)}$$

Indicators of Deprivation

Weights of different indicators (Determined by EU rules)

Rules that apply in period t. Change every 7 years; e.g. in 2000

Instrument construction

Conditional IV

$$\Delta \ln Unemp = \beta \Delta NGE + \beta D_{pre2000} + \nu$$

$$\Delta NGE = \sum_k D_{k,2002} W_{k,post2000} - D_{k,1997} W_{k,pre2000}$$

$$\Delta Z = \sum_k D_{k,pre2000} W_{k,post2000} - D_{k,pre2000} W_{k,pre2000}$$

After controlling for $D_{k,pre2000}$, ΔZ does no longer depend on ν and thus becomes valid instrument

Implementation

```
df=read.csv( "https://www.dropbox.com/s/8pdffaq268v7m8o/unempprep.csv?dl=1")
 # Simple OLS
     regOLS=lm(  DDDln1Punemp ~ DDDNGE ,df)
   summary(regOLS)
##
## Call:
## lm(formula = DDDln1Punemp ~ DDDNGE, data = df)
##
## Residuals:
##     Min       1Q   Median       3Q      Max
## -3.5451 -0.1907  0.0165  0.2109  2.8601
##
## Coefficients:
##              Estimate Std. Error  t value Pr(>|t|)
## (Intercept) -0.462220   0.003648 -126.703  < 2e-16 ***
## DDDNGE      -0.221062   0.036012   -6.138 8.62e-10 ***
```

A change in support from 0 to 10% (0-0.1) will reduce unemployment by 2.2%

IV

List of historic deprivation controls

```r
controls=c("gdp91","manufshare_1991","popdens_1981","current_unemprate1991
","actrate_1991","resid_emp_rate92")
fff=paste(controls, collapse  ="+")
```

ΔZ

Now run iv:

```r
 fffiv  =paste0( "DDDln1Punemp ~ DDDNGE+     ",fff, "| DDDxnivav +",fff)
 regIV=ivreg(  fffiv ,data=df)
 summary(regIV)
```

A change in support from 0 to 10% (0-0.1) will reduce unemployment by 4.5%

```
##
## Call:
## ivreg(formula = fffiv, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.52984 -0.18444  0.01339  0.20575  2.74181
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)            -0.7851275  0.0820543  -9.568  < 2e-16 ***
## DDDNGE                 -0.4546888  0.1131435  -4.019 5.89e-05 ***
## gdp91                   0.0030923  0.0003235   9.559  < 2e-16 ***
## manufshare_1991         0.5714105  0.0481121  11.877  < 2e-16 ***
## popdens_1981            0.0004555  0.0004222   1.079    0.281
## current_unemprate1991  -2.1341897  0.4972815  -4.292 1.79e-05 ***
## actrate_1991            0.4900480  0.0628717   7.794 7.07e-15 ***
## resid_emp_rate92       -0.3915193  0.0876794  -4.465 8.08e-06 ***
## ---
```

First Stage

Check first stage

```
ffffs  =paste0( "DDDNGE~DDDxnivav+     ",fff)

regFS=lm(  ffffs ,df)
summary(regFS)
```

```
##
## Call:
## lm(formula = ffffs, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.38214 -0.01537  0.00344  0.02657  0.43048
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)            -2.210e-01  2.303e-02  -9.599  < 2e-16 ***
## DDDxnivav               1.015e+00  2.932e-02  34.627  < 2e-16 ***
## gdp91                  -4.575e-04  8.145e-05  -5.617 2.00e-08 ***
## manufshare_1991        -1.038e-01  1.146e-02  -9.059  < 2e-16 ***
## popdens_1981            1.969e-04  1.070e-04   1.839  0.06592 .
## current_unemprate1991   1.061e+00  1.372e-01   7.732 1.16e-14 ***
## actrate_1991           -4.786e-02  1.592e-02  -3.007  0.00265 **
## resid_emp_rate92        3.674e-01  2.482e-02  14.803  < 2e-16 ***
## ---
```

The IV variable is called DDDxnivav

F-test of Z=0

```
linearHypothesis(regFS,"DDDxnivav=0")
```

```
## Linear hypothesis test
##
## Hypothesis:
## DDDxnivav = 0
##
## Model 1: restricted model
## Model 2: DDDNGE ~ DDDxnivav + gdp91 + manufshare_1991 +
popdens_1981 +
##     current_unemprate1991 + actrate_1991 + resid_emp_rate92
##
##   Res.Df     RSS Df Sum of Sq    F    Pr(>F)
## 1  10757 104.652
## 2  10756  94.156  1    10.496 1199 < 2.2e-16 ***
## ---
```

F statistic is larger 10…hurray!!!

Reduced Form

Reduced Form:

```r
fffrf  =paste0( "DDDln1Punemp~DDDxnivav+      ",fff)
summary(lm(  fffrf ,df))
```

```
##
## Call:
## lm(formula = fffrf, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.5300 -0.1834  0.0138  0.2023  2.7342
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)             -0.6846189  0.0898471  -7.620 2.75e-14 ***
## DDDxnivav               -0.4616909  0.1144068  -4.036 5.49e-05 ***
## gdp91                    0.0033003  0.0003178  10.386  < 2e-16 ***
## manufshare_1991          0.6185964  0.0446947  13.840  < 2e-16 ***
## popdens_1981             0.0003659  0.0004176   0.876    0.381
## current_unemprate1991 -2.6164480  0.5352012  -4.889 1.03e-06 ***
## actrate_1991             0.5118111  0.0621070   8.241  < 2e-16 ***
## resid_emp_rate92        -0.5585603  0.0968272  -5.769 8.21e-09 ***
## ---
```

IF reduced form is not significant IV will not be either

Panic on the streets of London
(Study by Draca, Machin and Witt)

Does more police on the street lead to less crime?

Simple regression?

$$Crime_{Area} = \beta Policetime_{Area} + \epsilon_{Area}$$

Police go where the crime is

Panic on the streets of London

(Study by Draca, Machin and Witt)

Solution:

Use

July 7 bombings of 2005 led to increases in police in certain areas (central London, near tube stations) which had nothing to do with crimes such as pick pocketing etc.

Results: 10% more police activity reduces crime by 3 to 4 percent.

Panic on the streets of London

$Crime_{Post 7/7} - Crime_{Pre\ 7/7}$

$$\Delta Crime_a = \beta \Delta Police_a + \epsilon_a$$

$Z_a = area\ \ Close\ to\ terror\ target$

Summary

- Endogeneity is often a problem: X(ϵ)
- However, X is also driven by other factors
- If we can find data on at least on other factor Z which is independent of ϵ we can do 2SLS IV
- Can combine with using various other controls to make it more plausible that remaining error ϵ is indeed independent of Z
- Need to ensure strong first stage
- Finding IVs is a bit of an art