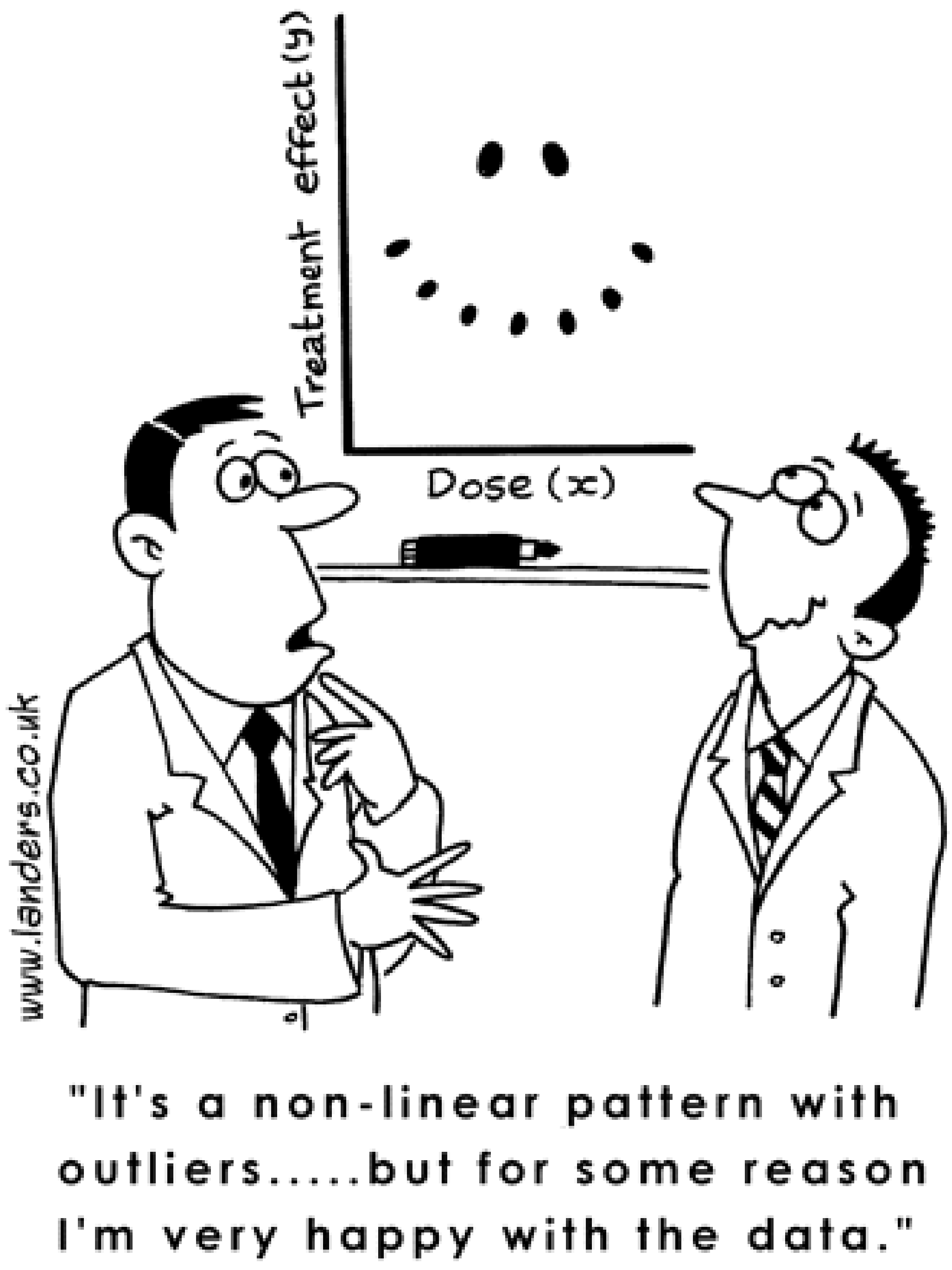

Imperial means
Intelligent Business

Econometrics for dummies

About qualitative and nonlinear relationships

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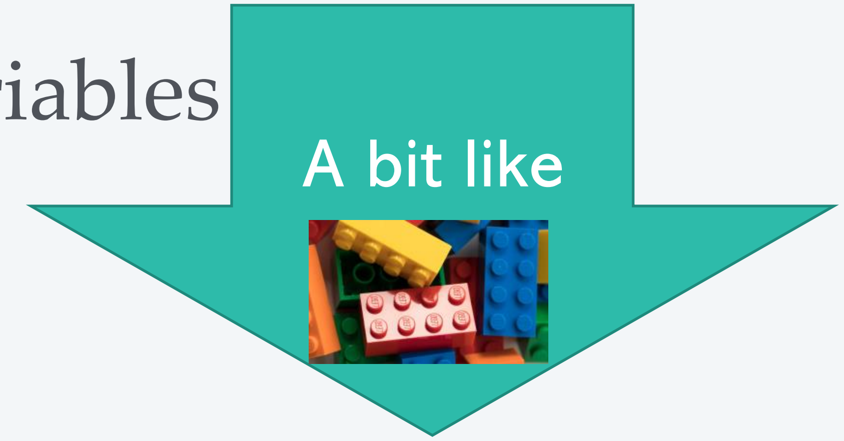
Objectives for this lecture

- Learn how to deal with modelling qualitative aspects of reality

- We can code those with dummy (binary) variables
  - As explanatory variables
  - As dependent variables



- Appreciate that dummy variables an important building block for constructing more sophisticated models
  - Allowing for non-linear relationships
  - Controlling for many potential confounding factors

- There are some other easy example of nonlinear models. Let’s look at those.

Looking at wage regressions as example again:

```r
wage1 <-  read.csv("https://www.dropbox.com/s/9agc2vmamfztlel/WAGE1.csv?dl=1")


r1 <- lm(wage ~ female, wage1)
r1 %>% summary()
```

```
##
## Call:
## lm(formula = wage ~ female, data = wage1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.5995 -1.8495 -0.9877  1.4260 17.8805
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.0995     0.2100  33.806  < 2e-16 ***
## female       -2.5118     0.3034  -8.279 1.04e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.476 on 524 degrees of freedom
## Multiple R-squared:  0.1157, Adjusted R-squared:  0.114
## F-statistic: 68.54 on 1 and 524 DF,  p-value: 1.042e-15
```

Change the x variable by 1 unit. What's the effect on the Y variable?

i.e. go from male to female, what's the effect on wages?

How can we interpret the coefficients?

$$Wage = \beta_0 + \beta_1 \times FEMALE + \epsilon$$

A closer look at those regression coefficients

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.0995     0.2100  33.806  < 2e-16 ***
## female       -2.5118     0.3034  -8.279 1.04e-15 ***
## ---
```

```
wage1 %>% group_by(female) %>% summarize(mean(wage))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 2 x 2
##   female `mean(wage)`
##    <int>        <dbl>
## 1      0         7.10
## 2      1         4.59
```

Average wage

Notice, that the intercept (β_0) is the average wage for men. To get the average wage for women we need to add the coefficient:

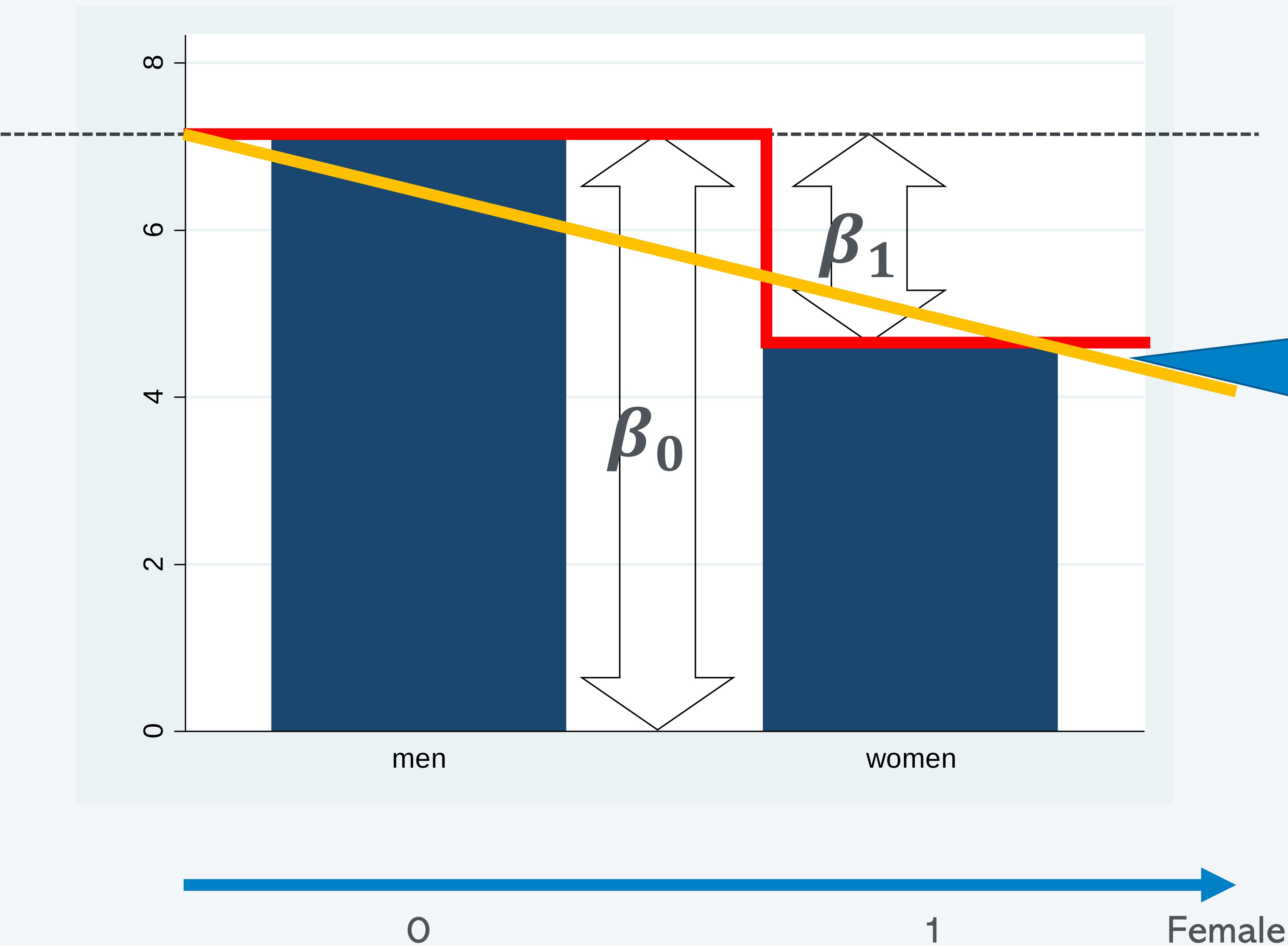
$$E\{wage|Women\} = \beta_0 + \beta_1 = 4.587...$$

where $E\{wage|Women\}$ is a mathematical way of saying the average (or expected) wage for women.

Conditional expectation

Dummies as bars

The underlying model: $Y = \beta_0 + \beta_1 FEMALE + \epsilon$



Instead of a linear relationship between the x and y variable we we get a nonlinear one

What about men?

```
> summary(lm(wage ~ male +female, wage1))

Call:
lm(formula = wage ~ male + female, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.5995 -1.8495 -0.9877  1.4260 17.8805 

Coefficients: (1 not defined because of singularities)
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   4.5877     0.2190  20.950  < 2e-16 ***
male          2.5118     0.3034   8.279 1.04e-15 ***
female            NA         NA      NA       NA    
```

- R dropped "female" because of multi-collinearity

- **We cannot see from our data what happens when female changes while male is kept constant.**

- Interpretation of coefficients: $E\{Y|Women\} = \beta_{Const}$

  $E\{Y|Men\} = \beta_{Cons} + \beta_{male}$

  i.e. $\beta_{male} = E\{Y|Men\} - E\{Y|Women\}$

2 cases in the data. We only need 2 parameters to represent those

Another option

```
> summary(lm(wage ~ 0+male +female, wage1))

Call:
lm(formula = wage ~ 0 + male + female, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.5995 -1.8495 -0.9877  1.4260 17.8805 

Coefficients:
       Estimate Std. Error t value Pr(>|t|)    
male      7.099      0.210   33.81   <2e-16 ***
female    4.588      0.219   20.95   <2e-16 ***
---
```

- Instead of dropping “female” we can drop “male”
- However, we can also drop the constant as in the regression above
- Consequently $E\{Y|Women\} = \beta_{female}$
  $E\{Y|Men\} = \beta_{male}$

Main take-away

- Various ways to represent the same thing/model that men and women have a different average wage by including combinations of dummy variables from the following
  - "constant" : always equal to 1
  - "male" : equal to 1 for men
  - "female" : equal to 1 for women

- Which dummies we include exactly will affect the interpretation of the coefficients ($\beta' s$)

- If we include "constant" and "male" ("female") then "female" ("male") becomes the **reference category**

- The mean of the reference category is represented by the constant coefficient

Sets of dummies

- Dummies "male" and "female" classify the sample exhaustively
- We often have classifications with more than two categories
- e.g. rather than having the education in years we might have only a categorical variable capturing 3 levels of education:

```
wage1= wage1 %>% mutate(educats=0,
                        educats=ifelse(educ==12,1,educats),
                        educats=ifelse(educ>12,2,educats))

wage1 %>% group_by(educats) %>% summarize(n())
```

```
## # A tibble: 3 x 2
##   educats `n()`
##     <dbl> <int>
## 1       0   116
## 2       1   198
## 3       2   212
```

Your turn: How would you conduct regression analysis of the relationship between wage and education if all you had was the educats variable?

(a) e.g. lm(wage~educats)
(b) something else?

Regression on categorical variable

```
> summary(lm(wage ~ educats, wage1))

Call:
lm(formula = wage ~ educats, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.2933 -2.2542 -0.9292  1.2301 17.6867 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   3.8751     0.2765  14.016   <2e-16 ***
educats       1.7091     0.1961   8.717   <2e-16 ***
```

Can you interpret this coefficient?

Creating sets of dummies from categories approach 1

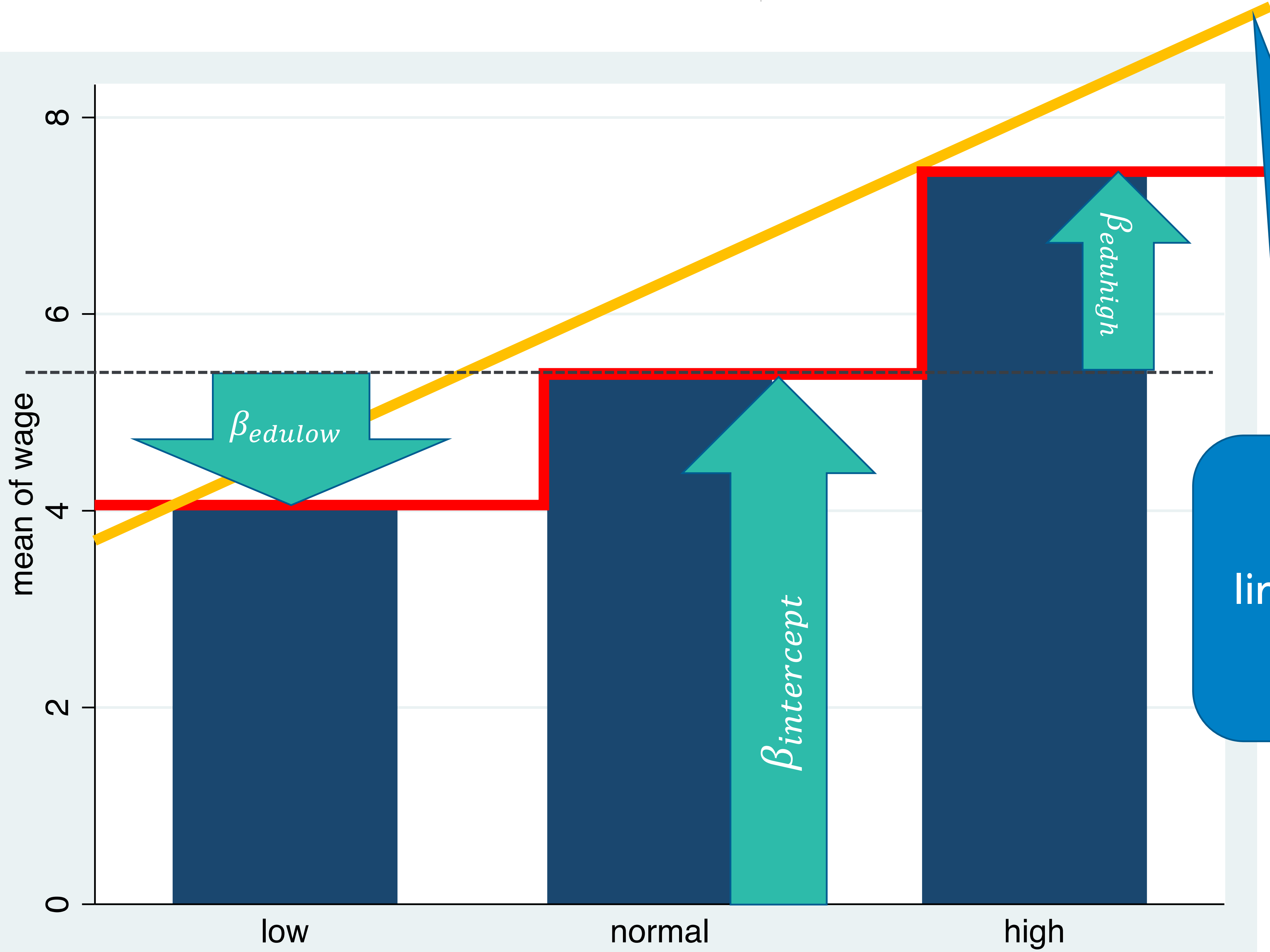
```
wage1=wage1 %>% mutate(edu_low=educats==0,
                       edu_normal=educats==1,
                       edu_high=educats==2)

reg=lm(wage~edu_low+edu_normal+edu_high,wage1)

reg %>% summary()
```

```
##
## Call:
## lm(formula = wage ~ edu_low + edu_normal + edu_high, data = wage1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.393 -2.119 -1.033  1.245 17.587
##
## Coefficients: (1 not defined because of singularities)
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)      7.3926     0.2372  31.165  < 2e-16 ***
## edu_lowTRUE     -3.3359     0.3989  -8.363 5.56e-16 ***
## edu_normalTRUE  -2.0213     0.3413  -5.922 5.78e-09 ***
## edu_highTRUE         NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

- We cannot regress dummies for all categories and a constant (dummy variable trap)
- R makes sure we don't fall in the trap and drops one of the dummies (thank you R…that was a close call)

Interpretation

```
## Coefficients: (1 not defined because of singularities)
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)        5.3714     0.2454  21.884  < 2e-16 ***
## edu_lowTRUE       -1.3146     0.4038  -3.255  0.00121 **
## edu_highTRUE       2.0213     0.3413   5.922 5.78e-09 ***
## edu_normalTRUE         NA         NA      NA       NA
```



Non-linear step function....however linear relationship looks like a fairly good approximation

Testing the validity of a linear model

Linear: A change of 1 in x variable always has the same effect on the y variable

This means in current context

Edu low to mid = Edu mid to high

How to say that in terms of model parameters?

- β_{mid} , β_{high} $\beta_{mid} = \frac{\beta_{high}}{2}$

$$Wage = \beta_0 + \beta_{edulow}\text{edu\_low} + \beta_{eduhigh}\text{edu\_high} + \epsilon$$

Test $\beta_{edulow} = -\beta_{eduhigh}$

Testing the validity of the linear model

```r
reg=lm(wage ~ edu_low+edu_high, wage1)

linearHypothesis(reg,c("edu_lowTRUE=-edu_highTRUE"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## edu_lowTRUE  + edu_highTRUE = 0
##
## Model 1: restricted model
## Model 2: wage ~ edu_low + edu_high
##
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    524 6253.5
## 2    523 6238.6  1    14.888 1.2481 0.2644
```

What do you conclude?

High p-value, so we cannot reject. Hence, it would be valid to use the linear model here

Creating sets of dummies from categories approach 2

- Instead of creating a separate dummy variable for every category, we can tell R that we are dealing with a categorical/factor variable
- Good if we have a large number of categories

```
wage1 =wage1 %>% mutate(educatsf=factor(educats,label=c("low","normal","high")))
```

R creates dummy variables automatically

```
> summary(lm(wage ~ educatsf, wage1))

Call:
lm(formula = wage ~ educatsf, data = wage1)

Residuals:
   Min     1Q Median     3Q    Max
-5.393 -2.119 -1.033  1.245 17.587

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      4.0567     0.3207  12.651  < 2e-16 ***
educatsfnormal   1.3146     0.4038   3.255  0.00121 **
educatsfhigh     3.3359     0.3989   8.363 5.56e-16 ***
---
```

```
> summary(lm(wage ~ 0+educatsf, wage1))

Call:
lm(formula = wage ~ 0 + educatsf, data = wage1)

Residuals:
   Min     1Q Median     3Q    Max
-5.393 -2.119 -1.033  1.245 17.587

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
educatsflow      4.0567     0.3207   12.65   <2e-16 ***
educatsfnormal   5.3714     0.2454   21.88   <2e-16 ***
educatsfhigh     7.3926     0.2372   31.16   <2e-16 ***
---
```

Several Dummy Sets

```
> wage1["gender"]<-factor(wage1$female, label=c("male","female"))
> summary(lm(wage ~ educatsf+gender, wage1))

Call:
lm(formula = wage ~ educatsf + gender, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.4467 -2.0765 -0.4759  0.9779 16.6133 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)      5.1207     0.3280  15.611  < 2e-16 ***
educatsfnormal   1.6052     0.3817   4.206 3.06e-05 ***
educatsfhigh     3.2460     0.3755   8.644  < 2e-16 ***
genderfemale    -2.3735     0.2867  -8.278 1.06e-15 ***
---
```

- Implied model: $Wage = \beta_C$
  $$+\beta_{normal} normal + \beta_{high} high$$
  $$+\beta_{female} female + \epsilon$$
- Reference category for education: Low
- Reference category for gender: Male

Interpretation

$$Wage = \underline{\beta_C} + \beta_{normal} normal + \beta_{high} high + \beta_{female} female + \epsilon$$

$\beta_C = E\{WAGE|Educ\ low, Male\}$

Interpretation

$$Wage = \beta_C + \beta_{normal} normal + \beta_{high} high$$
$$+\beta_{female} female + \epsilon$$

$E\{WAGE|Educ\ Low, Female\} = \beta_C + \beta_{female}$

$E\{WAGE|Educ\ Normal, Female\} = \beta_C + \beta_{female} + \beta_{normal}$

Dummies as dependent variables

- So far we discussed dummies as explanatory variables
- However, we might also have dummies as dependent variables

- E.g Bertrand Mullainathan Data:

```
bm=read.csv("https://www.dropbox.com/s/e4w6pjm6b4wz9do/bm.csv?dl=1")
```

- We regress $CALL = \beta_0 + \beta_1 BLACK + \epsilon$

$$CALL = \begin{cases} 1 \text{ call back} \\ 0 \text{ otherwise} \end{cases}$$

- Hence, following the discussion in this lecture:
- $\beta_0 = E\{CALL|Non\ Black\}$

Average of the CALL variable for Non Black

- $\beta_1 = E\{CALL|Black\} - E\{CALL|Non\ Black\}$

Difference between average for Black vs Non Black

Interpretation of β's with Dummy as dependent variable

- $\beta_0 = E\{CALL|Non\ Black\} = \frac{\sum_{i \in NonBlack} CALL_i}{n_{NonBlack}} = P\{Call|NonBlack\}$

- Average CALL foris the sum of CALL divided by number of Non Black people in the sample
- Because CALL is either 0 or 1 this is equivalent to the share that received a call back
- Which we can think of as an estimate of the call back probability

- $\beta_1 = E\{CALL|Black\} - E\{CALL|Non\ Black\}$
  $= P\{Call|Black\} - P\{Call|NonBlack\}$

- $\hat{\beta}_0$: share of non Black receiving call back
- $\hat{\beta}_1$: share of black receiving call - of share of non-Black receiving call

- i.e. there is a natural interpretation of coefficients when regressing dummies on dummies

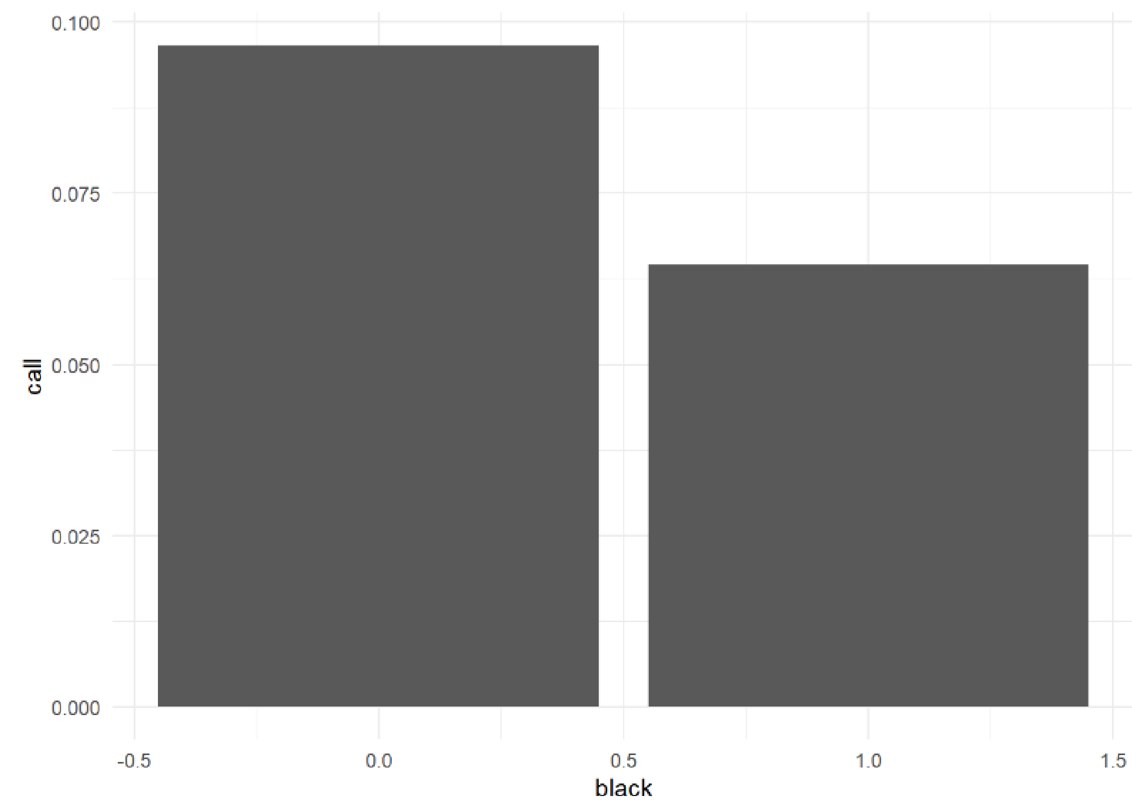
- Things are a bit less clear when regressing dummies on – say – a linear term

Dummies as dependent variables - In action

```
summary(lm(call~black,bm))
```

```
##
## Call:
## lm(formula = call ~ black, data = bm)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.09651 -0.09651 -0.06448 -0.06448  0.93552
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.096509   0.005505  17.532  < 2e-16 ***
## black       -0.032033   0.007785  -4.115 3.94e-05 ***
## ---
## Signif. codes:  0 '***     001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error:          4868 degrees of freedom
## Multiple R-squared:  0.0034          ted R-squared:  0.003261
## F-statistic: 16.93 on 1 and             ue: 3.941e-05
```

```
library(ggplot2)
agg=bm %>% group_by(black) %>% summarise(call=mean(call))

ggplot(agg,aes(x=black,y=call))+geom_bar(stat="identity")+theme_m
inimal()
```



CVs with “black” sounding names have a 3.2 **percentage point** lower chance of receiving a call back

Dummies as dependent variables - linear model case

$$Call = \beta_1 + \beta_2 Experience + \epsilon$$

```
(linear0 <- lm(call ~ yearsexp        , data = bm)) %>% summary()
```

```
##
## Call:
## lm(formula = call ~ yearsexp, data = bm)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.20030 -0.08101 -0.07439 -0.06776  0.94218
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0545046  0.0071949   7.575 4.26e-14 ***
## yearsexp    0.0033136  0.0007716   4.295 1.78e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2716 on 4868 degrees of freedom
## Multiple R-squared:  0.003774,   Adjusted R-squared:  0.00357
## F-statistic: 18.44 on 1 and 4868 DF,  p-value: 1.784e-05
```

Probability of call back increases by 0.3 percentage points with every additional year of experience

Dummies as dependent variables…and many dummies as explanatory variables as an alternative to a linear relationship

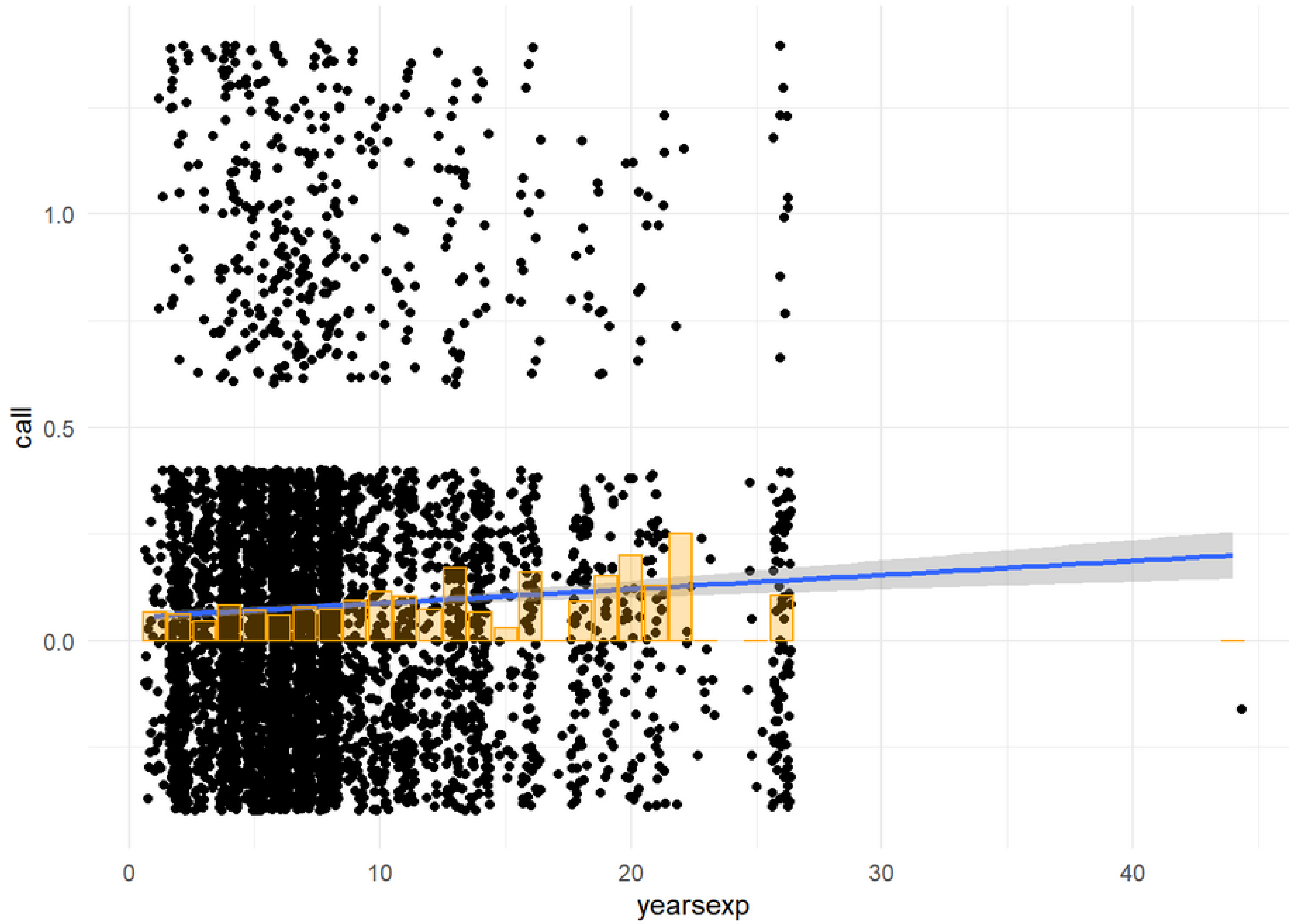
```
(linear1 <- lm(call ~ factor(yearsexp), data = bm)) %>% summary()
```

```
##
## Call:
## lm(formula = call ~ factor(yearsexp), data = bm)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.25000 -0.08194 -0.07246 -0.05998  0.97059
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          0.0666667  0.0404239   1.649   0.0992 .
## factor(yearsexp)2   -0.0041667  0.0429301  -0.097   0.9227
## factor(yearsexp)3   -0.0202749  0.0448679  -0.452   0.6514
## factor(yearsexp)4    0.0152700  0.0420835   0.363   0.7167
## factor(yearsexp)5    0.0043393  0.0421797   0.103   0.9181
## factor(yearsexp)6   -0.0066911  0.0415222  -0.161   0.8720
## factor(yearsexp)7    0.0109673  0.0420715   0.261   0.7943
## factor(yearsexp)8    0.0059977  0.0419680   0.143   0.8864
## factor(yearsexp)9    0.0276730  0.0457883   0.604   0.5456
## factor(yearsexp)10   0.0487179  0.0469013   1.039   0.2990
## factor(yearsexp)11   0.0373796  0.0453778   0.824   0.4101
## factor(yearsexp)12   0.0057971  0.0519596   0.112   0.9112
## factor(yearsexp)13   0.1021645  0.0459520   2.223   0.0262 *
## factor(yearsexp)14   0.0004474  0.0461260   0.010   0.9923
## factor(yearsexp)15  -0.0372549  0.0616186  -0.605   0.5455
## factor(yearsexp)16   0.0929078  0.0491566   1.890   0.0588 .
## factor(yearsexp)17  -0.0666667  0.1616955  -0.412   0.6801
## factor(yearsexp)18   0.0242424  0.0508830   0.476   0.6338
## factor(yearsexp)19   0.0855072  0.0568565   1.504   0.1327
## factor(yearsexp)20   0.1333333  0.0611152   2.182   0.0292 *
## factor(yearsexp)21   0.0609929  0.0565566   1.078   0.2809
## factor(yearsexp)22   0.1833333  0.1040473   1.762   0.0781 .
## factor(yearsexp)23  -0.0666667  0.0990179  -0.673   0.5008
## factor(yearsexp)25  -0.0666667  0.1101769  -0.605   0.5451
## factor(yearsexp)26   0.0391026  0.0483854   0.808   0.4190
## factor(yearsexp)44  -0.0666667  0.2741681  -0.243   0.8079
## ---
```

Alternatively use a model with a separate effect for every year of experience; e.g. with 13 years the probability goes up by 10 percentage points relative to reference group

Effect of experience on call back

```
agg2=agg=bm %>% group_by(yearsexp) %>% summarise(call=mean(call))
ggplot(bm,aes(x=yearsexp,y=call) ) + geom_jitter()+
           geom_smooth(method="lm")+
           geom_bar(data=agg2,aes(x=yearsexp,y=call),
                    stat="identity",color="orange",
                    fill="orange",alpha=0.3)+
          theme_minimal()
```



Other non-linear relationships

- Relationship between explanatory and dependent variables may be non-linear

- There are general methods to deal with this
- However, in many cases we can avoid using different methods because many types of seemingly non-linear relationship can be represented in what boils down to a linear regression.
- e.g. suppose you suspect that the relationship between wage and education in wage1.dta is actually following a quadratic form:

$$Wage = \beta_0 + \beta_1 EDU + \beta_2 EDU^2 + \epsilon$$

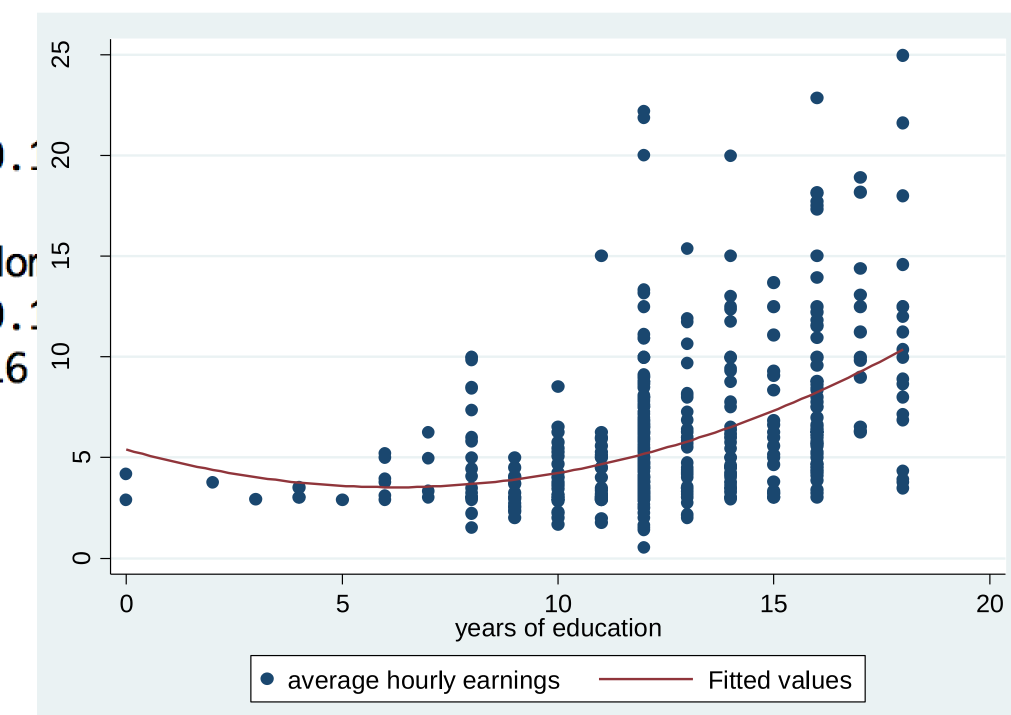
Your turn: Any ideas how to deal with this?

A square relationship


```
wage1=wage1 %>%
mutate(educ2 =educ^2)
```

```
>  summary(lm(wage ~ educ+educ2, wage1))

Call:
lm(formula = wage ~ educ + educ2, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.8722 -2.0002 -0.7472  1.2642 17.0159 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  5.40769    1.45886   3.707 0.000232 ***
educ        -0.60750    0.24149  -2.516 0.012181 *  
educ2        0.04907    0.01007   4.872 1.46e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 **’ 0.01 ‘*’ 0.05 ‘.’ 0.

Residual standard error: 3.  on 523 degrees of freedor
Multiple R-squared:  0.201      Adjusted R-squared:  0.
F-statistic: 65.79 on 2 a     DF,  p-value: < 2.2e-16
```

Seems to be significant



How to interpret things?

Note what we do in the linear case $Y = \beta X + \epsilon$

$$\frac{\partial Y}{\partial X} = \beta$$

In the linear case β is the marginal effect of X on Y

We can work out the same thing in the nonlinear case

$$Y = \beta_1 X + \beta_2 X^2 + \epsilon \longrightarrow \frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X$$

- *The marginal effect (how much Y changes in response to change in X) varies for different values of X*
- *We can also find the extreme point by looking at* $\frac{\partial Y}{\partial X} = 0$

log-linear relationships

- The most popular non-linear model is probably

$$Y = \exp(\beta_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \epsilon)$$

- To make it linear all that is required is to take the (natural) logarithm on both sides of the equation:

$$\ln Y = \beta_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \epsilon$$

- One of the reasons why it's popular is the interpretation of the β coefficients it implies

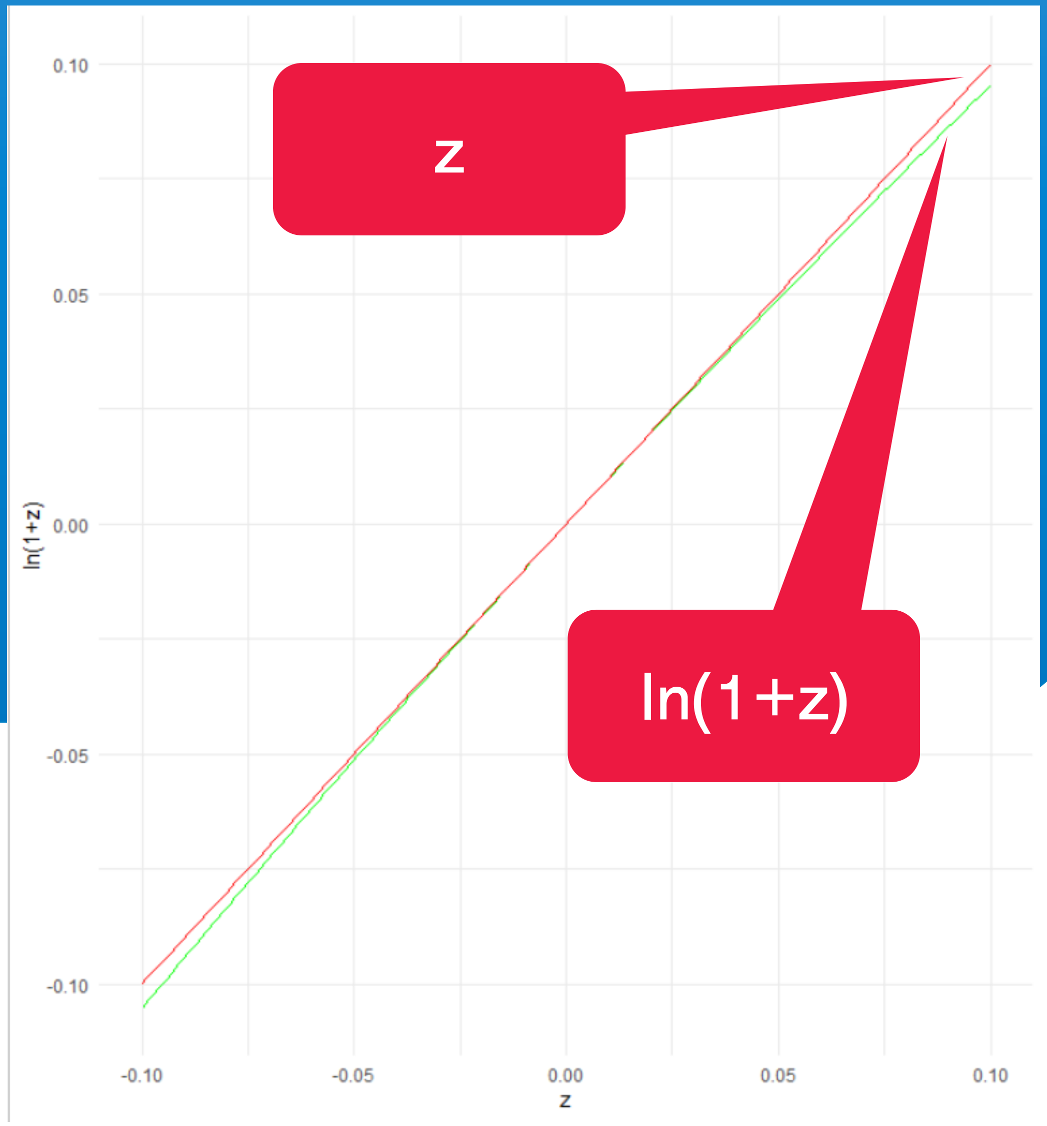
Interpreting log-linear relationships

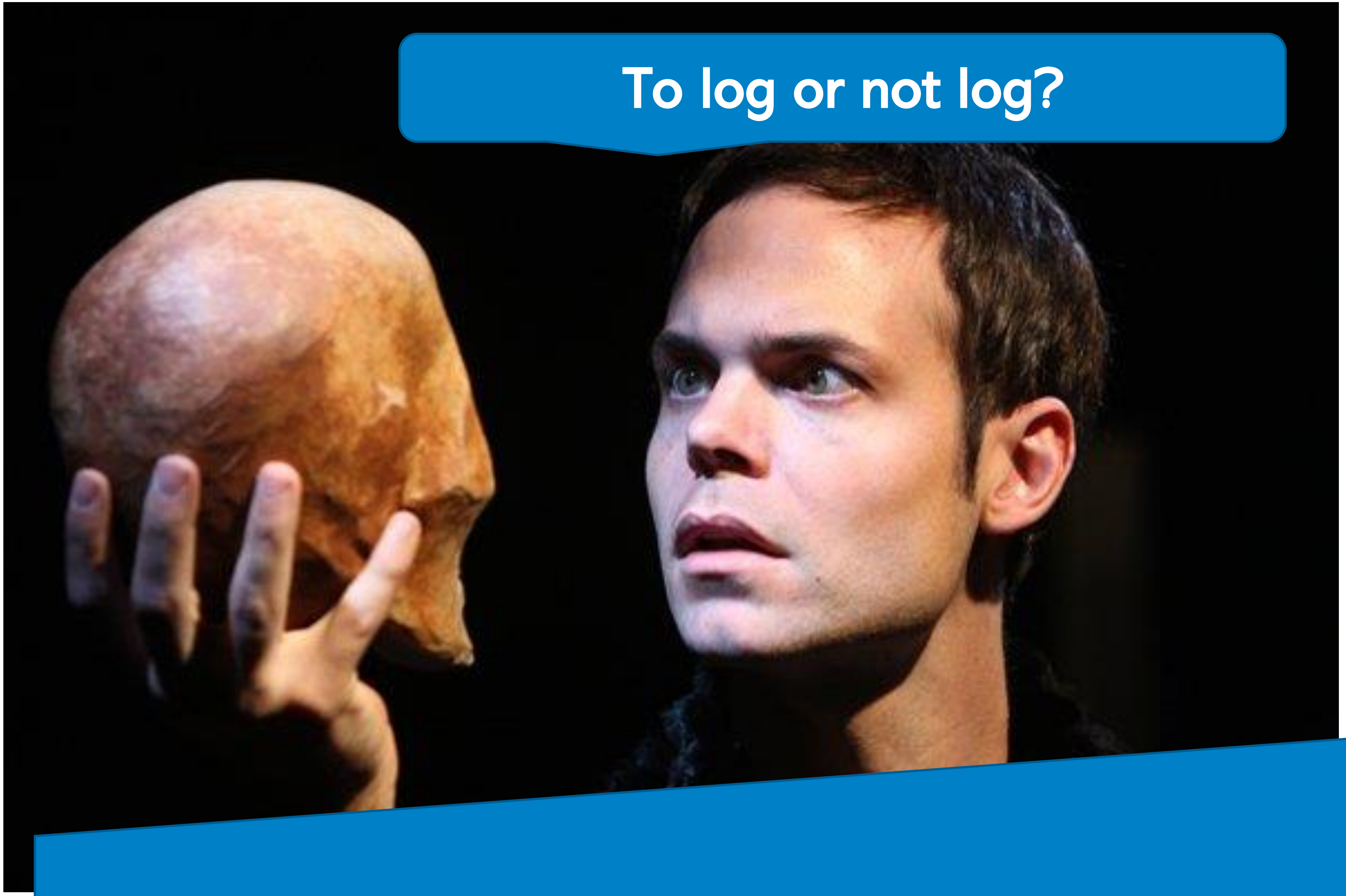
$$\ln Y = \cdots + \beta X + \epsilon$$

- As before: change in dependent variable for given change in X
- However now:
- $DepVar_a - DepVar_b$
- $= \ln Y_a - \ln Y_b$
- $= \ln\left(\frac{Y_a}{Y_b}\right)$
- $= \ln\left(1 + \frac{Y_a}{Y_b} - 1\right)$
- $\approx \frac{Y_a}{Y_b} - 1$
- $= \frac{Y_a - Y_b}{Y_b}$

Let $z = \frac{Y_a}{Y_b} - 1$

Hence, β captures (approximately) the Growth in dependent variable Y when we change X by 1 unit





Which is more plausible?

1. Change in X leads to fixed change in Y→ use Y
2. Change in X leads to fixed percentage change in Y→ use $\ln Y$

Going log crazy: log log

$$\ln Y = \cdots + \beta \quad \ln X \quad + \epsilon$$

- As before: change in dependent variable for given change in X
- However now:

$$\beta = \frac{DepVar_a - DepVar_b}{XVar_a - XVar_b} = \frac{\ln Y_a - \ln Y_b}{\ln X_a - \ln X_b}$$

$$= \frac{\frac{Y_a - Y_b}{Y_b}}{\frac{X_a - X_b}{X_b}}$$

Elasticity

Famous example: production functions

Cobb Douglas production function:

Value added

$$Y = AL^{\alpha_L} K^{\alpha_K}$$

employment

capital

Productivity shock: $A_0 \exp(\epsilon)$

Taking logs:

$$\ln Y = \alpha_0 + \alpha_L \ln L + \alpha_K \ln K + \epsilon$$

Elasticity of output with respect to a change in employment

log transformation overview

Case	Formula	Interpretation of β coefficient
No logs	$y = \beta x + \epsilon$	Change of 1 unit of X leads to β units of Y
log on LHS	$\ln y = \beta x + \epsilon$	Change of 1 unit of X leads to $\beta \times 100\ \%$ growth in Y
log on RHS	$y = \beta \ln x + \epsilon$	A 1% growth of X leads to a $\frac{\beta}{100}$ change of Y in units of Y
log on RHS & LHS	$\ln y = \beta \ln x + \epsilon$	A 1% growth of X leads to $\beta\%$ growth in Y

Summary

- Dummies are cool: Model qualitative and non-linear aspects
- Don't fall in the dummy variable trap
- The same model can be represented in several ways
- Be careful with interpretation of dummies
- A lot of stuff that looks non-linear at first glance is linear after all

Extra Slides

Interactions?

In the context of the model regressing wages on skill and gender we might ask if income gap between man and women is the same irrespective of the educational group; e.g.

$$E\{Wage|Women, Low\} - E\{Wage|Men, Low\}$$

$=E\{Wage|Women, Normal\} - E\{Wage|Men, Normal\}$?

Notice that in terms of the model we used before it is impossible to have differences in the gap across education group.

$$Wage = \beta_C + \beta_{normal} normal + \beta_{high} high + \beta_{female} female + \epsilon$$

By construction the gap is always β_{female}

Interactions?

In the context of the model regressing wages on skill and gender we might ask if income gap between man and women is the same irrespective of the educational group; e.g.

$= \beta_{female}$

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$$=E\{Wage|Women, Normal\} - E\{Wage|Men, Normal\}?$$

$= \beta_{female}$

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$$Wage = \beta_C + \beta_{normal}normal + \beta_{high}high + \beta_{female}female + \epsilon$$

By construction the gap is always β_{female}

A more complex model

We can get a more complex model using interactions:

$$Wage = \beta_C + \beta_{normal} normal + \beta_{high} high$$

$$+\beta_{female} female$$

$$+\beta_{fem \times norm} normal \times female$$

$$+\beta_{fem \times high} high \times female \; + \epsilon$$

A more complex model

We can get a more complex model using interactions:

$$E\{Wage|Women, Low\} - E\{Wage|Men, Low\} = \beta_{female}$$

$$E\{Wage|Women, Normal\} - E\{Wage|Men, Normal\} = \beta_{female} + \beta_{fem \times norm}$$

How would you test if the wage gap is different for normally educated persons?

Interactions?

```
> summary(lm(wage ~ educatsf*gender, wage1))

Call:
lm(formula = wage ~ educatsf * gender, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.5235 -1.8394 -0.4407  0.9131 16.5365 

Coefficients:
                            Estimate Std. Error t value Pr(>|t|)    
(Intercept)                   4.6780     0.4057  11.530  < 2e-16 ***
educatsfnormal                2.2683     0.5371   4.223 2.85e-05 ***
educatsfhigh                  3.7656     0.4989   7.548 1.99e-13 ***
genderfemale                 -1.3859     0.6059  -2.287   0.0226 *  
educatsfnormal:genderfemale  -1.3737     0.7644  -1.797   0.0729 .  
educatsfhigh:genderfemale    -1.1749     0.7567  -1.553   0.1211    
---
```

How does the wage gap differ for different educational groups?

Interactions and continuous variables

```
> summary(lm(wage ~ educatsf*exper, wage1))

Call:
lm(formula = wage ~ educatsf * exper, data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.3796 -1.9595 -0.6658  1.3310 16.3972 

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)          3.468072   0.501029   6.922 1.32e-11 ***
educatsfnormal       1.294190   0.650598   1.989  0.04720 *  
educatsfhigh         2.469483   0.617135   4.002 7.21e-05 ***
exper                0.028100   0.018721   1.501  0.13396    
educatsfnormal:exper 0.005466   0.026468   0.206  0.83648    
educatsfhigh:exper   0.077254   0.027425   2.817  0.00503 ** 
---
```

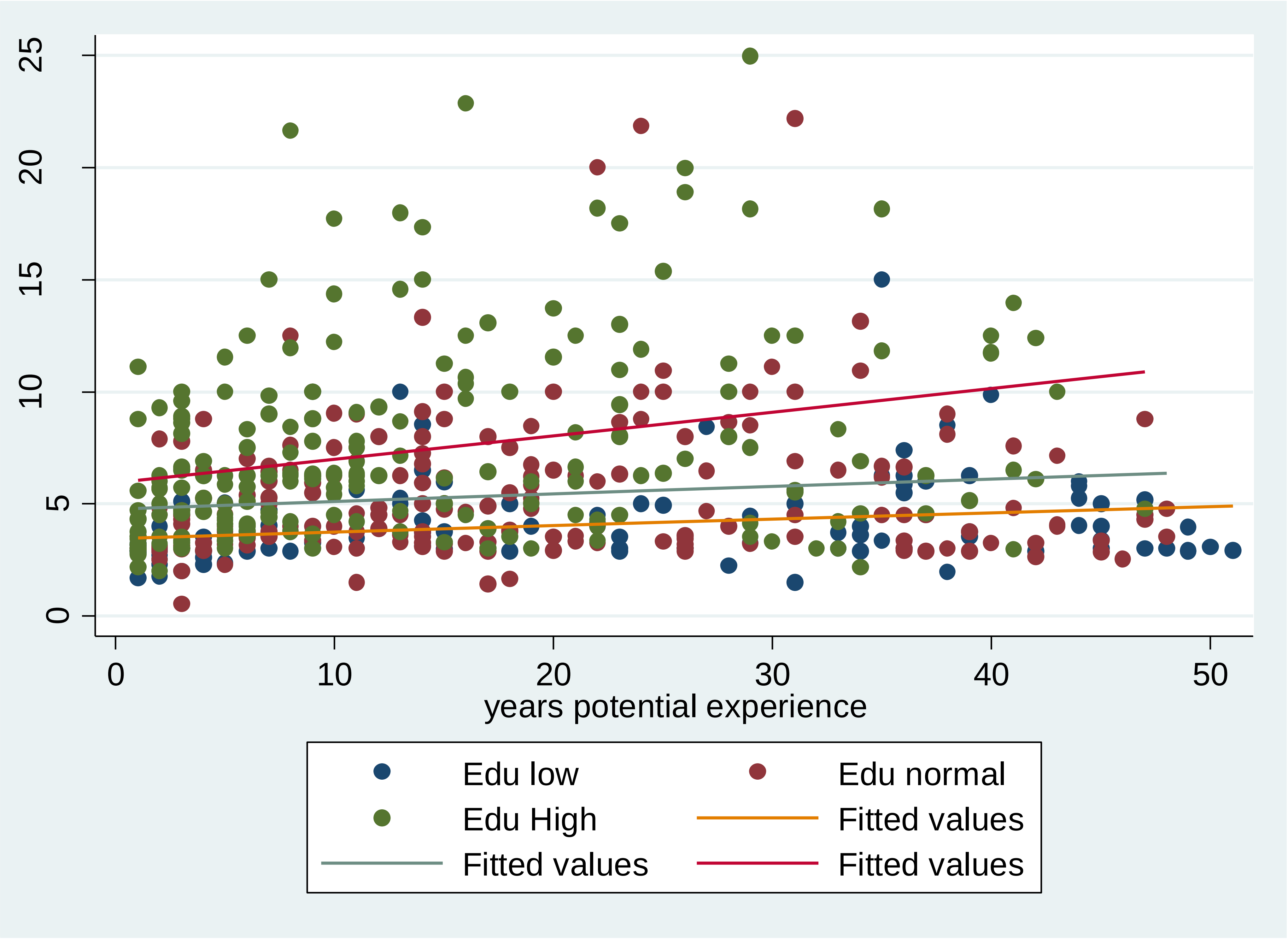
Your turn: Does experience affect the educational groups differently?

(a) No

(b) Yes: experience has a bigger impact for the highest educated

(c) Yes: experience has a smaller impact for the highest educated

Interactions and continuous variables



Relating the figure to the model

The overall model is:

$$Wage = \beta_C + \beta_{normal} normal + \beta_{high}\, high + \beta_{exper} exper$$

$$+\beta_{normal \times exper}\, normal \times exper \; + \beta_{high \times exper}\, high \times exper + \epsilon$$

Topline (EDU high): $\beta_C + \beta_{high} + \left(\beta_{exper} + \beta_{high \times exper}\right) \times exper$

Middle line (EDU normal): $\beta_C + \beta_{normal} + \left(\beta_{exper} + \beta_{normal \times exper}\right) \times exper$

Bottom line (EDU low): $\beta_C + \beta_{exper} \times exper$

So many dummies

- Note that we often use large numbers of dummies in regression models
- Helps to control for a wide range of potential confounding factors

Accounts for sector level confounding factors

Examples:

- Sector dummies when using firm level data from several sectors
- Regional dummies
- Firm dummies when having panel data for firms
- Time dummies when having panel data

Firm panel data example

Filter

	id	year	va	l	k	lnk	lnl	lnva
1	72	79	6420.9556	15	2402.043	7.784075	2.708050	8.767322
2	72	80	3684.0342	15	2602.932	7.864394	2.708050	8.211764
3	72	81	3936.5615	22	2606.353	7.865707	3.091042	8.278063
4	72	82	7624.0674	16	2335.772	7.756098	2.772589	8.939065
5	72	83	3699.7642	13	2102.915	7.651080	2.564949	8.216024
6	72	84	3526.3325	22	1901.783	7.550547	3.091042	8.168014
7	72	85	2918.3970	19	1727.427	7.454388	2.944439	7.978790
8	72	86	4707.2549	23	1575.752	7.362488	3.135494	8.456860
9	81	79	18382.6211	29	43056.445	10.670267	3.367296	9.819161
10	81	80	27541.7852	32	46128.199	10.739180	3.465736	10.223460
11	81	80	31956.9297	32	43702.344	10.685157	3.465736	10.372144
12	81	82	54484.9492	34	42768.898	10.663567	3.526361	10.905680
13	81	83	42444.5078	32	41514.715	10.633803	3.465736	10.655953
14	81	84	60194.0078	31	39277.289	10.578402	3.433987	11.005328
15	81	85	43816.1484	33	35558.312	10.478930	3.496508	10.687758
16	81	86	32128.8516	39	33107.188	10.407506	3.663562	10.377510

Firm panel data example

```r
library(dplyr)
prod=read.csv("https://www.dropbox.com/s/v1j4xzkaado8zlz/prod_b
alanced.csv?dl=1")
prod=prod %>% mutate(lnl=log(l),lnva=log(va))

reg1<-lm(lnva~lnl+lnk,prod)
reg1 %>% summary()
```

```
##
## Call:
## lm(formula = lnva ~ lnl + lnk, data = prod)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.8415 -0.4453  0.0326  0.4820  3.0979
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.423770   0.034256   99.95   <2e-16 ***
## lnl         0.950352   0.010352   91.81   <2e-16 ***
## lnk         0.304229   0.005801   52.45   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
```

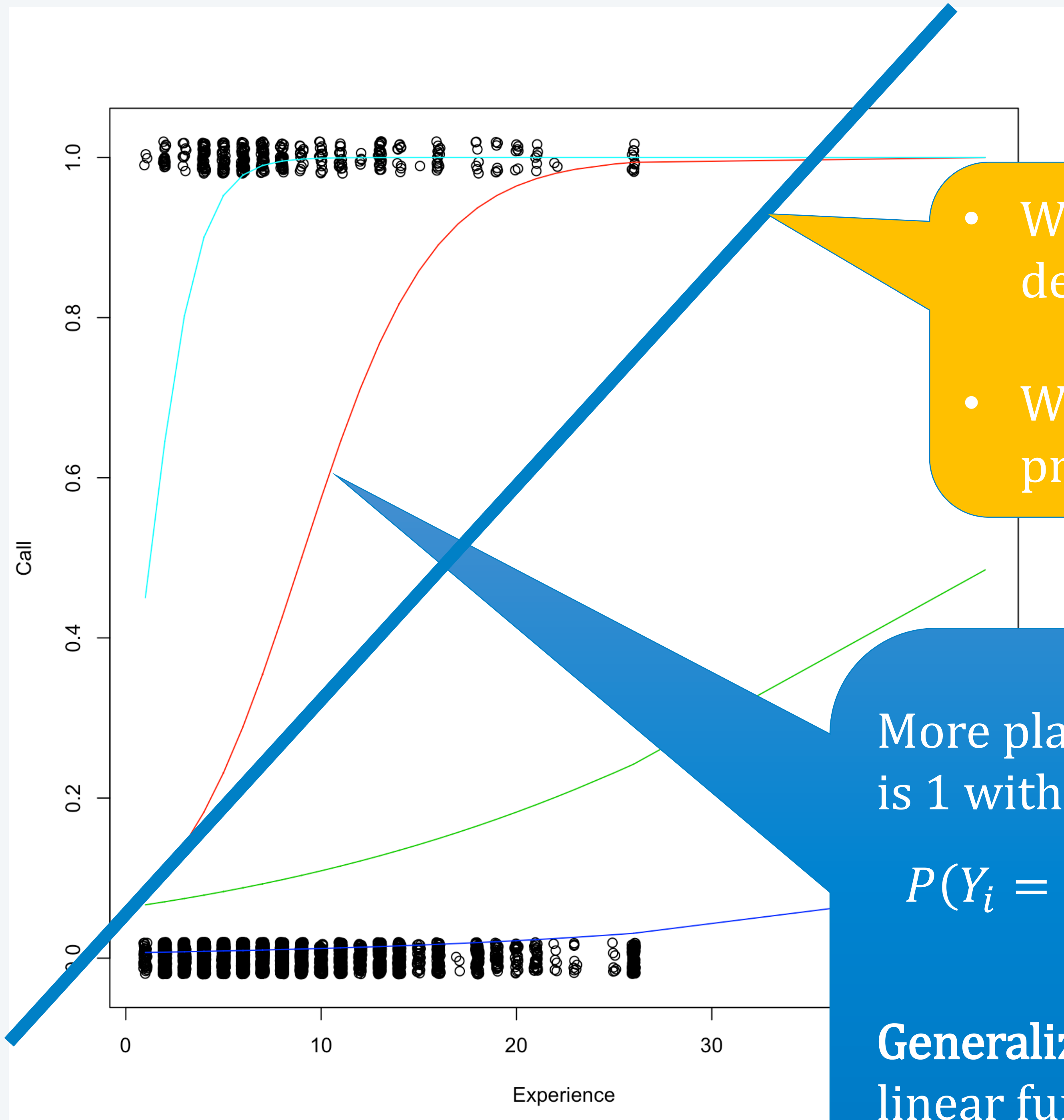
```r
reg2<-lm(lnva~lnl+lnk+factor(year)+factor(id),prod)
reg2 %>% summary()
```

```
##
## Call:
## lm(formula = lnva ~ lnl + lnk + factor(year) + factor(id), d
ata = prod)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.5554 -0.2349  0.0323  0.2767  3.2121
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      5.6053565  0.
## lnl              0.7302864  0.
## lnk              0.0832028  0.
## factor(year)80   0.0775959  0.
## factor(year)81   0.1594889  0.
## factor(year)82
## factor(year)83  -0.042
## factor(year)84   0.0061375  0.
## factor(year)85  -0.0543233  0.0
## factor(year)86   0.0725690  0.0173079   4.193 2.77e-05 ***
## factor(id)81     1.4392760  0.2608948   5.517 3.52e-08 ***
## factor(id)99    -0.1671771  0.2580364  -0.648 0.517073
## factor(id)117    0.8002776  0.2582686   3.099 0.001948 **
## factor(id)135   -0.0069199  0.2578253  -0.027 0.978588
## factor(id)162    1.5912130  0.2632387   6.045 1.54e-09 ***
## factor(id)171    1.0838099  0.2594566   4.177 2.97e-05 ***
## factor(id)198   -0.6712341  0.2578931  -2.603 0.009258 **
## factor(id)342    0.5866663  0.2610479   2.247 0.024635 *
## factor(id)396    0.1227912  0.2579268   0.476 0.634033
```

Controlling for year effects: e.g. a recession could depress sales (and thus value added) and investment

Controlling for firm effects: successful firms will attract more investment and will have higher sales (and thus value added)

Revisiting dummies as dependent variables



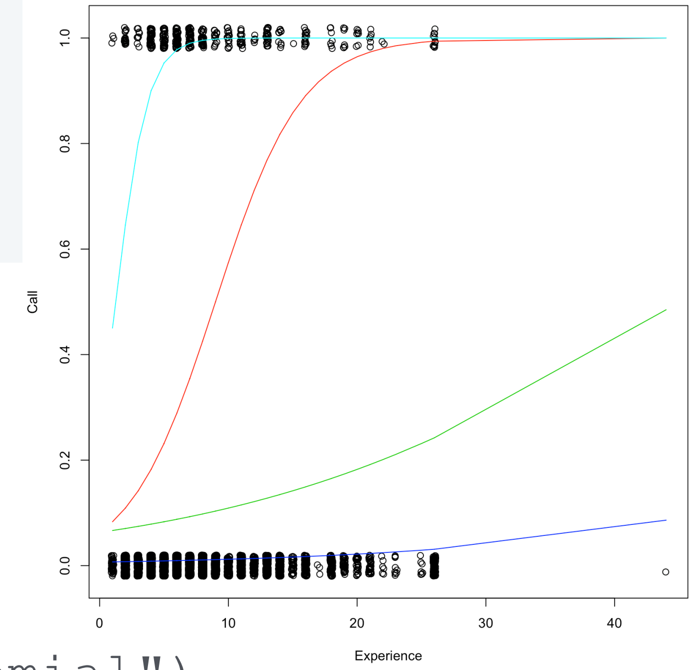
- We saw we could model the probability of the dependent variable equal to 1 as a line
$$P(Y_i = 1|X_i) = \beta_0 + \beta_1 X$$
- We could interpret β_1 as the change in probability in response to change in X

More plausible model probability that outcome dummy is 1 with the logistic function:

$$P(Y_i = 1|X_i) = \frac{1}{1 + \exp(-[\beta_0 + \beta_1 X])} = g(\beta_0 + \beta_1 X)$$

Generalized linear model: A linear model inside a non-linear function

Implement logit in R




```
> library(aod)
>   library(margins )
>   logit <- glm(call ~ yearsexp, data = bm, family = "binomial")
>   summary(logit)

Call:
glm(formula = call ~ yearsexp, family = "binomial", data = bm)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.7780  -0.4075  -0.3924  -0.3779   2.3598

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.75960    0.09620 -28.687  < 2e-16 ***
yearsexp     0.03908    0.00918   4.257 2.07e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2726.9  on 4869  degrees of freedom
Residual deviance: 2710.2  on 4868  degrees of freedom
```

How to interpret things?

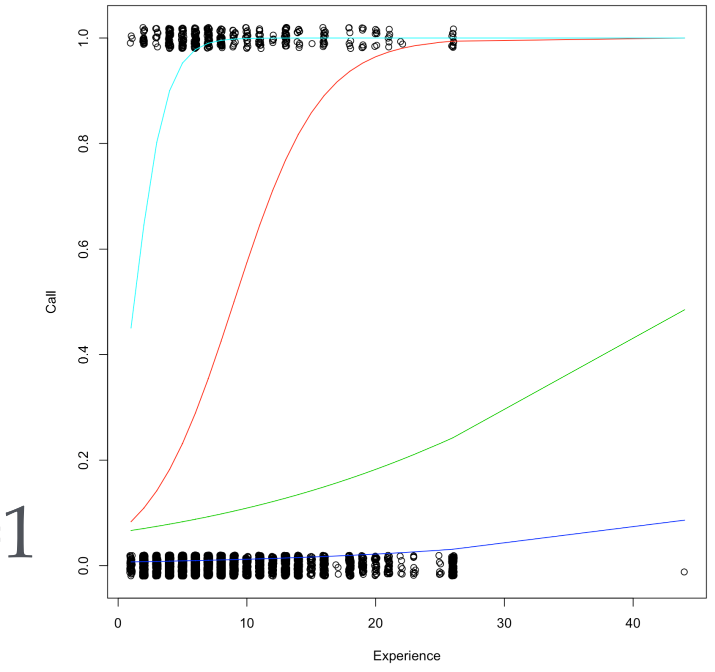
Note what we do in the linear case $Y = \beta X + \epsilon$

$$\frac{\partial Y}{\partial X} = \beta$$

In the linear case β is the marginal effect of X on Y

We can work out the same thing in the nonlinear case

How to interpret things?



Compute marginal effect on of variable on probability of Y=1

$$\frac{\partial P(Y_i = 1|X_i)}{\partial X_i} = \frac{exp(-\hat{\beta} X_i)\hat{\beta}}{\left[1 + exp(-\hat{\beta} X_i)\right]^2}$$

Not constant for different observations. So we have to compute for every observation separately. Or at specific observations we are interested in.
R margins() command will help

```
>   margins(logit)
Average marginal effects
glm(formula = call ~ yearsexp, fam

 yearsexp
 0.002881
```

Can be compared to linear probability model

```
## Coefficients:
##              Estimate Std. Error
## (Intercept) 0.0545046  0.0071949
## yearsexp    0.0033136  0.0007716
## ---
```